APPENDIX G3 Traffic Grade Crossing Delay Methodology

Appendix G3

Grade Crossing Delay Calculation Methodology Mathematical Derivation of Delay Equation

The methodology for computing vehicular delay is based on Figure G-1, which shows total vehicle arrivals and departures for an isolated grade crossing blockage. The yellow line represents vehicles arriving at an at-grade crossing, beginning at the time when the gates go down (point "O" in the figure). Total gate down time is depicted as "T_G". The green line represents the vehicles departing the queue after the gate is lifted starting at time = T_G (point "A" in the figure). The queues are fully dissipated at time = t* (point "B" in the figure). The total vehicle delay is represented by the area of triangle OAB bounded by the yellow line, the green line, and the "X" axis. The length of line $S = (t_2 - t_1)$ represents the amount delay experienced by the nth vehicle. Calculating the value of this line for each vehicle arriving at the crossing and then adding those values up is equivalent to computing the area of triangle OAB. This calculation is performed for each train arriving at the crossing over the course of a day. Delay will vary by time of day, because there is more highway traffic during peak hours.

The equation for total vehicle delay for an isolated blockage, V, is:

$$V = \left(\frac{1}{2}\right) \frac{qT_G^2}{(1-q/d)}$$

Note that delay is a function of the square of the gate down time.

The equation for the arrival line in the graphic, A(t), is:

y = qt

where:

y = cumulative number of vehicles arriving at the crossing

q = arrival rate in vehicles per minute

t = time in minutes

Arrivals are assumed to be uniformly distributed (i.e., vehicles arrive at equal intervals).

The equation for the departure line in the graphic, D(t), is:

$$y = d(t - T_G)$$

where:

y = cumulative number of vehicles departing the crossing after the gates are up

- *d*= departure rate in vehicles per minute
- T_G = gate down time in minutes



Figure G-1:Cumulative Arrivals and Departures for an Isolated Blockage

Source: Graphic and mathematical derivation adapted from Dr. Robert C. Leachman, San Pedro Bay Access Study: Phase 2: Railroad Access, 1984, Appendix G, Figure G-1, prepared for Southern California Association of Governments (SCAG). Original equations for computing vehicle hours of delay are from James Powell, Effects of Rail-Highway Grade Crossings on Highway Users, Presentation to the Transportation Research Board, January 19, 1982, p. 12.

The total number of vehicles (*N*) delayed by the train is calculated as follows:

 $N = qt^*$

where:

 t^* = time at which the queues have fully dissipated

Point B in Figure G-1 is where the arrival and departure lines intersect. The coordinates of point B are (t^*, N) . At this point,

 $qt^* = d(t^* - T_G)$

Thus,

$$t^* = \frac{dT_G}{d-q} = \frac{T_G}{(1-q/d)}$$

Since $N = qt^*$,

$$N = \frac{qT_G}{(1 - q/d)}$$

The number of vehicle minutes of delay (V) for an isolated blockage is derived by calculating the area of the triangle OAB.

$$V = \frac{1}{2}(t^*)(d)(t^* - T_G) - \frac{1}{2}(t^* - T_G)(d)(t^* - T_G)$$
$$= \left(\frac{1}{2}\right) \frac{qT_G^2}{(1 - q/d)}$$