

**Grade Crossing Delay Calculation Methodology
Including Mathematical Derivation of Delay Equation**

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The methodology for computing vehicular delay is based on Figure G-1, which shows total vehicle arrivals and departures for an isolated grade crossing blockage. The yellow line represents vehicles arriving at an at-grade crossing, beginning at the time when the gates go down (point "O" in the figure). Total gate down time is depicted as " T_G ". The green line represents the vehicles departing the queue after the gate is lifted starting at time = T_G (point "A" in the figure). The queues are fully dissipated at time = t^* (point "B" in the figure). The total vehicle delay is represented by the area of triangle OAB bounded by the yellow line, the green line, and the "X" axis. The length of line $S = (t_2 - t_1)$ represents the amount delay experienced by the nth vehicle. Calculating the value of this line for each vehicle arriving at the crossing and then adding those values up is equivalent to computing the area of triangle OAB. This calculation is performed for each train arriving at the crossing over the course of a day. Delay will vary by time of day, because there is more highway traffic during peak hours.

The equation for total vehicle delay for an isolated blockage, V , is:

$$V = \left(\frac{1}{2}\right) \frac{qT_G^2}{(1 - q/d)}$$

Note that delay is a function of the square of the gate down time.

Derivation of Formula

The equation for the arrival line in the Figure G-1, $A(t)$, is:

$$y = qt$$

where:

y = cumulative number of vehicles arriving at the crossing

q = arrival rate in vehicles per minute

t = time in minutes

Arrivals are assumed to be uniformly distributed (i.e., vehicles arrive at equal intervals).

The equation for the departure line in the graphic, $D(t)$, is:

$$y = d(t - T_G)$$

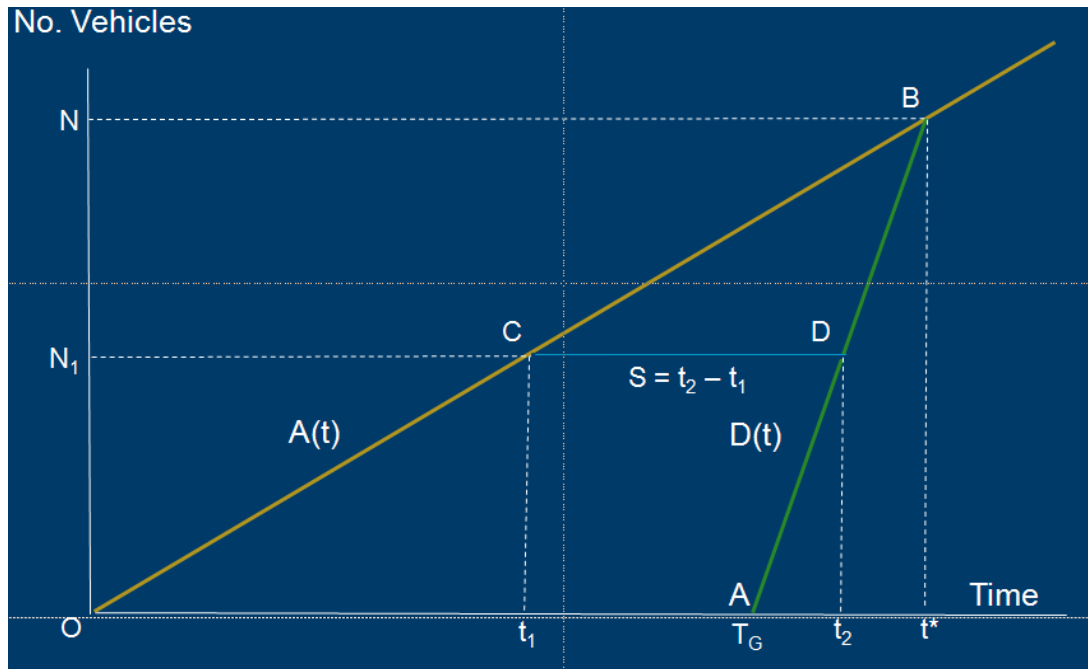
where:

y = cumulative number of vehicles departing the crossing after the gates are up

d = departure rate in vehicles per minute

T_G = gate down time in minutes

Figure G-1: Cumulative Arrivals and Departures for an Isolated Blockage



Source: Graphic and mathematical derivation adapted from Dr. Robert C. Leachman, San Pedro Bay Access Study: Phase 2: Railroad Access, 1984, Appendix G, Figure G-1, prepared for Southern California Association of Governments (SCAG). Original equations for computing vehicle hours of delay are from James Powell, Effects of Rail-Highway Grade Crossings on Highway Users, Presentation to the Transportation Research Board, January 19, 1982, p. 12.

The total number of vehicles (N) delayed by the train is calculated as follows:

$$N = qt^*$$

where:

t^* = time at which the queues have fully dissipated

Point B in Figure G-1 is where the arrival and departure lines intersect. The coordinates of point B are (t^*, N) . At this point,

$$qt^* = d(t^* - T_G)$$

Thus,

$$t^* = \frac{dT_G}{d - q} = \frac{T_G}{(1 - q/d)}$$

Since $N = qt^*$,

$$N = \frac{qT_G}{(1 - q/d)}$$

The number of vehicle minutes of delay (V) for an isolated blockage is derived by calculating the area of the triangle OAB.

$$\begin{aligned} V &= \frac{1}{2}(t^*)(d)(t^* - T_G) - \frac{1}{2}(t^* - T_G)(d)(t^* - T_G) \\ &= \left(\frac{1}{2}\right) \frac{qT_G^2}{(1 - q/d)} \end{aligned}$$

Application of Formula

This calculation is performed for each train arriving at the crossing over the course of a day. Delay will vary by time of day, because there is more highway traffic during peak hours. Many of the vehicles arriving at the crossing will not be delayed by a train, but they are included in the calculation of average delay. This is the same way that average delay is computed for signalized intersections.

Hourly average delay per vehicle is calculated by dividing total delay over one hour by the number of vehicles arriving at the crossing in the same hour.

The calculation of hourly average vehicle delay accounts for the following:

- Total vehicles arriving at the crossing in a one-hour period, whether the vehicles are delayed by a train or not.
- Total delay experienced by all vehicles in that hour.
- All freight and passenger trains passing through the crossing in that hour.

The delay equation above relates to the effects of an isolated blockage; i.e., it is assumed that the vehicle queues are completely dissipated before the next train arrives at the crossing. However, where the rail corridor has more than one track, it is possible that a second train traveling in the opposite direction could arrive at the crossing before the queues from the first train have fully dissipated. More complex delay equations for these “multiple events” have been derived by Dr. Robert Leachman of U.C. Berkeley.¹ In an effort to compute these effects and how likely they are to occur, Dr. Leachman simulated railroad traffic for both 2010 and 2035 against streets with varying ADT per lane and recomputed vehicular delays including the impacts of multiple

¹ Robert C. Leachman, *San Pedro Bay Access Study: Phase 2: Railroad Access*, 1984, Appendix G, prepared for Southern California Association of Governments (SCAG).

events. With higher train volumes, multiple events occur more often, and the severity of the impact is greater on streets with more vehicular traffic per lane. Based on a sample of Dr. Leachman's results for different train volumes and ADT per lane, Cambridge Systematics fitted a curve for the calculation of a "Bias Factor". This Bias Factor adjustment accounts for additional delay associated with multiple crossings that overlap in time. The fitted equation for the Bias Factor, BF, is as follows:

$$BF = \exp \left(-0.52868 + (.000173) \times \left(\frac{ADT}{Lane} \right) + (0.01036) \times (Total\ Train\ Volume\ per\ Day) \right)$$

The R-squared value for the fitted equation is 0.9322, indicating a very good correlation among the variables. Using this equation, a Bias Factor was computed for each grade crossing that has more than one track crossing the street. The Bias Factor is then multiplied by the unadjusted vehicle hours of delay for an isolated blockage to account for the effects of multiple events. For example, the average Bias Factor for all grade crossings on the BNSF San Bernardino Subdivision for 2027 is approximately 1.055, meaning that the unadjusted delay values are increased by an average of 5.5 percent.